

# Probability

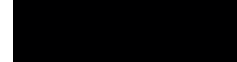
## 1. Properties of Probability

### (1) Terminology

- $\emptyset$  denotes the null or empty set;
- $A \subseteq B$  means A is a subset of B;
- $A \cup B$  is the union of A and B;
- $A \cap B$  is the intersection of A and B;
- $A^c$  is the complement of A (i.e., all elements in S that are not in A).
- Mutually exclusive and Exhaustive events:

$A_1, A_2, \dots, A_k$  are mutually exclusive events means that  $A_i \cap A_j = \emptyset, i \neq j$ ; that is,  $A_1, A_2, \dots, A_k$  are disjoint sets;

$A_1, A_2, \dots, A_k$  are exhaustive events means that  $A_1 \cup A_2 \cup \dots \cup A_k = S$



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(3) Venn diagrams

(4) Definition

Probability is a real-valued set function  $P$  that assigns, to each event  $A$  in the sample space  $S$ , a number  $P(A)$ , called the probability of the event  $A$ , such that the following properties are satisfied:

- a)  $P(A) \geq 0$ ;
- b)  $P(S) = 1$ ;
- c) if  $A_1, A_2, A_3, \dots$  are events and  $A_i \cap A_j = \emptyset$ ,  $i \neq j$ , then

$$P(A_1 \cup A_2 \cup \dots \cup A_k) = P(A_1) + P(A_2) + \dots + P(A_k)$$



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- d) For each event A,  $P(A) \leq 1$ .
- e) If A and B are any two events, then  $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ .
- f) If A, B, and C are any three events, then  $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$ .

## (1) Definition

The conditional probability of an event A, given that event B has occurred, is

defined by  $P(A|B) = \frac{P(A \cap B)}{P(B)}$ ,  $P(B) > 0$ .



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If  $A$  is an event, then  $A$  is the union of  $m$  mutually exclusive events, namely,

$$A = (B_1 \cap A) \cup (B_2 \cap A) \cup \dots \cup (B_m \cap A)$$

