Department of Mathematics
Comprehensive Examination - Option I
2019 Spring Semester

Topology

1. Let A and B be subsets of a topological space X. Prove: $\overline{A \cup B} = \overline{A} \cup \overline{B}$.

2. Let $f : X \to Y$ be a continuous bijective function from a compact space X to a Hausdorff space Y. Prove that f is a homeomorphism.

3. Let D be a space with discrete topology having at least two points. Prove: A topological space X is connected if and only if every continuous function $f : X \to D$ is constant.

4. Let X and Y be topological spaces and

$$C = \{U \times V : U \text{ is open in } X \text{ and } V \text{ is open in } Y\}$$

   (a) Prove: C forms a basis for a topology, the product topology, on $X \times Y$.

   (b) Let $\pi : X \times Y \to X$ be the projection map defined by $\pi(x, y) = x \quad \forall (x, y) \in X \times Y$. Use the tube lemma to prove that if Y is compact, then π is a closed map, i.e., $\pi(C)$ is closed in X for every closed set C in $X \times Y$.

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Algebra

1. Let G be a group and H be the subgroup generated by $\{x^{-1}y^{-1}xy \mid x, y \quad G\}$. Prove:
   (a) H is normal in G.
   (b) G/H is abelian.

2. Let Z be the ring of integers and suppose φ : Z $\to$ R is a homomorphism of Z onto the ring R. Prove that R is isomorphic as a ring either to Z_n for some positive integer n, or to Z.

3. Prove that each finite integral domain is a field; deduce that for each prime p, Z_p is a field.

4. Let V and W be vector spaces over a field; $\{x \quad V \mid \quad d(\quad 0)\}$ is a subspace f24.3V.8pover a31S883Td(955Tf24.3isomo)1. deduc78

   (b) T is one-to-one if and only if $N(\quad d) \quad \{0\}$ S of V, $S(L)$ is a hal
   f W T is one-to-one.

Department of Mathematics
Comprehensive Examination - Option III
2019 Spring Semester

Applied Analysis

1. Solve the linear system $X = AX$ with

$$A = \begin{pmatrix} -3 & 2 & -1 \\ 1 & -2 & 2 \\ 6 & -6 & 5 \end{pmatrix}$$ and initial condition X

4. A manufacturer of tribbles has warehouses in Atlanta, Baltimore, Chicago, and Detroit. The warehouse in Atlanta has 40 tribbles in stock, the warehouse in Baltimore has 70 tribbles in stock, the warehouse in Chicago has 40 tribbles in stock, and the warehouse in Detroit has 30 tribbles in stock. There are retail stores in Eugene, Fairview, Grove City, and Hayward that need to receive 50, 60, 30, and 40 tribbles, respectively. The following table gives the unit shipping costs from each warehouse to each retail store.

	Eugene	Fairview	Grove City	Hayward
Atlanta	10	15	20	18
Baltimore	20	22	18	15
Chicago	19	20	21	18
Detroit	25	30	28	29

Determine a shipping program that fulfills the required demands and minimizes the total shipping cost.

Department of Mathematics
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2019 Spring Semester

Probability

1. A salesperson has three clients in district A, five clients in district B, and seven clients in district C.

   (a) In how many ways can the salesperson select a group of clients so that in this group there are exactly two clients from each district?

   (b) In how many ways can the salesperson select six clients from these districts?

   (c) If the salesperson randomly selects six clients from these districts, then what is the probability that at least one of them is from district A?

   (d) If the salesperson randomly selects six clients from these districts and none of them is from district B, then what is the probability that exactly two of them are from district A?

   (e) If the salesperson randomly selects six clients from these districts and exactly three of them are from district B, then what is the probability that the salesperson selects the other three clients from district C?

2. Suppose that the cost X, in dollars, and the time Y, in minutes, of a job have the joint

4. The amount of bread (in hundreds of pounds) that a bakery is able to sell in one day is found to be a real-valued random variable X, with a probability density function given by

$$f(x) = \begin{cases} cx & \text{if } 0 \le x < 5 \\ c(10 - x) & \text{if } 5 \le x < 10 \\ 0 & \text{otherwise.} \end{cases}$$

   (a) Determine the value of c that makes $f(x)$ a density function.
   (b) Graph the density function.
   (c) What is the probability that the number of pounds of bread that will be sold tomorrow is
      i. more than 500 pounds?
      ii. less than 500 pounds?
      iii. between 250 pounds and 750 pounds?
   (d) Let A be the event that more than 500 pounds of bread is sold in a day; let B be the event that less than 500 pounds of bread is sold in a day; and let C be the event that between 250 pounds and 750 pounds of bread is sold in a day.
      i. Find $P(A|B)$.
      ii. Find $P(A|C)$.
      iii. Are events A and B independent? Justify your answer.
      iv. Are events A and C independent? Justify your answer.