Department of Mathematics and Computer Science

Comprehensive Examination–Option I

2015 Spring

Complex Analysis

1. Find a linear fractional transformation which maps the half plane $\{a + bi \mid a > b\}$ onto the disk $\{a + bi \mid a^2 + b^2 < 4\}$, and determine its inverse.

2. Find all solutions $x + iy$ of the equation $\sin z = 2i$.

3. Let f be a function analytic in an open set $A \subset \mathbf{C}$. Prove that if Re f is constant in A, then f is constant in A.

4. Let f be an entire function and $a, b \in \mathbf{C}$ with $|a| < R$ and $|b| < R$.

   a. Prove:
   $$f(a) - f(b) = \frac{a - b}{2\pi i} \int_{|z|=R} \frac{f(z)\,dz}{(z - a)(z - b)}$$

   b. Use part a. to prove Liouville's theorem: each bounded entire function is constant.

Department of Mathematics and Computer Science

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Topology

1. Let $B_l = \{[a, b) \mid a, b \in \mathbb{R} \text{ and } a < b\}$ be a collection of subsets of $\mathbb{R}$.

   (a) Prof $\mathbb{R}$

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Comprehensive Examination–Option III

2015 Spring

Applied Analysis

1. Find the general solution of the following initial value problem.

$$X' = \begin{pmatrix} 5 & -1 \\ 9 & -1 \end{pmatrix} X, \qquad X(0) = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

2. Use power series to find the solution about the point $x_0 = 0$ of the following equation. Give at least three nonzero terms of the series solution. Also, determine

Department of Mathematics and Computer Science

Comprehensive Examination–Option III

2015 Spring

Linear Programming

1. **Use the Primal Simplex method to solve the following minimization problem.**

$$\begin{array}{rrcrcrcl} \text{minimize} & -4x_1 & + & 5x_2 & - & 6x_3 & + & 3 \\ \text{subject to} & 3x_1 & + & 4x_2 & + & 6x_3 & \le & 8 \\ & -x_1 & + & 3x_2 & + & 2x_3 & \ge & 4 \\ & 2x_1 & + & 2x_2 & + & x_3 & \ge & 4 \\ & & & & & x_1, x_2, x_3 & \ge & 0 \end{array}$$

2. **Consider the following minimization problem.**

$$\begin{array}{rrcrcrcr} \text{minimize} & 9x_1 & + & 8x_2 & + & 8x_3 & & \\ \text{subject to} & 5x_1 & + & 3x_2 & + & 3x_3 & \le & 9 \\ & x_1 & + & 2x_2 & + & x_3 & \le & 4 \\ & 6x_1 & & & + & 4x_3 & \ge & 11 \\ & & & & & x_1, x_2, x_3 & \ge & 0 \end{array}$$

The beginning and final tableaux in the Simplex method are given in the following table.

	x_1	x_2	x_3	x_4	x_5	x_6	
x_4	5	3	3	1	0	0	9
x_5	1	2	1	0	1	0	4
a_1	6	0	4	0	0	−1	11
	9	8	8	0	0	0	0
x_1	1	6	0	2	0	3/2	3/2
x_5	0	5	0	1	1	1	2
x_3	0	−9	1	−3	0	−5/2	1/2
	0	26	0	6	0	13/2	−35/2

For each of the following scenarios return to the original problem. Use sensitivity analysis to answer the questions.

(a) **What is the range on the coefficient of x_1 such that the basis variables do not change?**

(b) **What is the range on the coefficient of x_2 such that the basis variables do not change?**

(c) **What is the range on the right hand side, $b_3 = 11$, of the third constraint such that the basis variables do not change?**

Linear Programming–continued

3. Solve the following transportation problem and give the final cost. The supplies are listed along the left, and the demands are listed along the top.

	50	75	100	125
55	29	10	22	11
100	31	18	32	16
55	32	35	37	33
65	28	12	20	17
75	30	19	30	15

4. Consider the following maximization problem.

$$\begin{aligned}
\text{maximize} \quad & 4x_1 + 4x_2 + 3x_3 \\
\text{subject to} \quad & 2x_1 - 2x_2 + x_3 \leq 10 \\
& 6x_1 + 3x_2 - x_3 \leq 16 \\
& 3/2\, x_1 + 4x_2 + 3x_3 \leq 15 \\
& -2x_1 + 6x_2 + 2x_3 \leq 10 \\
& x_j \geq 0
\end{aligned}$$

Using the Complementary Slackness Theorem prove or disprove the following statement.

$$\frac{42}{13}, 0, \frac{44}{13} \quad \text{is an optimal solution.}$$

Department of Mathematics and Computer Science

Comprehensive Examination–Option III

2015 Spring

Probability

Department of Mathematics and Computer Science

Comprehensive Examination–Option III

2015 Spring

Probability–continued

4. Company A has just developed a diagnostic test for a certain disease. The disease afflicts 1% of the population. The *sensitivity* of the test is the probability of someone testing positive, given that he or she has the disease, $P(+|D)$, and the *specificity* of the test is the probability that someone tests negative, given that he or she does not have the disease, $P(-|D^c)$. Assume that the sensitivity and specifity are each 95%.

   Company B, which is a rival of Company A, offers a competing test for the disease. Company B claims that their test is faster and less expensive to perform than the test from Company A, is less painful, and yet has a higher overall success rate, where the overall success rate is defined as the probability that a randomly selected person is diagnosed correctly.

   (a) The test from Company B can be described and performed very simply: no matter who the patient is, diagnose that he or she does not have the disease. Check whether the claim of Company B about overall success rates is true.

      i. Compute $P(D|+)$ and $P(D^c|-)$ for Company A.

      ii. Compute $P(D|+)$ and $P(D^c|-)$ for Company B.

      iii. Compare.

   (b) Explain why the test from Company A may still be useful.

   (c) Company A wants to develop a new test such that the overall success rate is higher than that of Company B.

      i. If the sensitivity and specificity are equal, how high does the sensitivity (t)3.44647(h)1.48798(a)-5.88723(t)-38